EXERCISE – V JEE PROBLEMS

1. Find the equations of the common tangents of the circle $x^2 + y^2 - 6y + 4 = 0$ and the parabola $y^2 = x$. **[REE 99, 6]**

Sol.

2. (a) If the line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$, then one of the values of 'k' is
(A) 1/8 (B) 8 (C) 4 (D) 1/4

Sol.

(b) If $x + y = k$ is normal to $y^2 = 12x$, then 'k' is **[JEE 2000 (Scr.), 1+1]**
(A) 3 (B) 9 (C) – 9 (D) – 3

Sol.

3. Find the locus of the points of intersection of tangents drawn at the ends of all normal chords of the parabola $y^2 = 8(x - 1)$. **[REE 2001, 3]**

Sol.

4. (a) The equation of the common tangent touching the circle$(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$ above the x-axis is

(A) $\sqrt{3}\, y = 3x + 1$ (B) $\sqrt{3}\, y = -(x + 3)$

(C) $\sqrt{3}\, y = x + 3$ (D) $\sqrt{3}\, y = -(3x + 1)$

Sol.

(b) The equation of the directrix of the parabola, $y^2 + 4y + 4x + 2 = 0$ **[JEE 2001 (Scr.), 1+1]**

(A) $x = -1$ (B) $x = 1$ (C) $x = -3/2$ (D) $x = 3/2$

Sol.

5. The locus of the mid-point of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with directrix **[JEE 2002 (Scr.), 3]**

(A) $x = -a$ (B) $x = -a/2$ (C) $x = 0$ (D) $x = a/2$

Sol.

6. The equation of the common tangent to the curves $y^2 = 8x$ and $xy = -1$ is **[JEE 2002 (Scr.), 3]**

(A) $3y = 9x + 2$ (B) $y = 2x + 1$

(C) $2y = x + 8$ (D) $y = x + 2$

Sol.

7. (a) The slope of the focal chords of the parabola $y^2 = 16x$ which are tangents to the circle $(x - 6)^2 + y^2 = 2$ are **[JEE 2003, (Scr.)]**

(A) ± 2 (B) – 1/2, 2 (C) ± 1 (D) – 2, 1/2

Sol.

(b) Normals are drawn from the point 'P' with slopes m_1, m_2, m_3 to the parabola $y^2 = 4x$. If locus of P with $m_1m_2 = a$ is a part of the parabola itself then find a. **[JEE 2003, 4]**

Sol.

8. The angle between the tangents drawn from the point (1, 4) to the parabola $y^2 = 4x$ is

(A) p/2 (B) p/3 (C) p/4 (D) p/6 **[JEE 2004, (Scr.)]**

Sol.

9. Let P be a point on the parabola $y^2 - 2y - 4x+5=0$, such that the tangent on the parabola at P intersects the directrix at point Q. Let R be the point that divides the line segment PQ externally in the ratio $\frac{1}{2}$: 1. Find the locus of R. **[JEE 2004, 4]**

Sol.

10. (a) The axis of parabola is along the line y = x and the distance of vertex from origin is $\sqrt{2}$ and that of origin from its focus is $2\sqrt{2}$. If vertex and focus both lie in the 1st quadrant, then the equation of the parabola is **[JEE 2006, 3]**

(A) $(x + y)^2 = (x - y - 2)$ (B) $(x - y)^2 = (x + y - 2)$

(C) $(x - y)^2 = 4(x + y - 2)$ (D) $(x - y)^2 = 8(x + y - 2)$

Sol.

(b) The equations of common tangents ot the parabola $y = x^2$ and $y = -(x - 2)^2$ is/are **[JEE 2006, 5]**

(A) $y = 4(x - 1)$ (B) $y = 0$

(C) $y = -4(x - 1)$ (D) $y = -30x - 50$

Sol.

(c) Match The Following **[JEE 2006, 6]**

Normals are drawn at point P, Q and R lying on the parabola $y^2 = 4x$ which intersect at (3, 0). Then

(i)	Area of DPQR	(A) 2
(ii)	Radius of circumcircle of DPQR	(B) 5/2
(iii)	Centroid of DPQR	(C) (5/2, 0)
(iv)	Circumcentre of DPQR	(D) (2/3, 0)

Sol.

11. Statement-1: The curve $y = \frac{-x^2}{2} + x + 1$ is symmetric with respect to the line x = 1.

Statement -2: A parabola is symmetric about its axis.

[JEE 2007, 4]

(A) Statement-1 is true, statement-2 is true ; statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true ; statement-2 is NOT a correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false
(D) Statement-1 is false, statement-2 is true

Sol.

Comprehension

12. Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in the first and the fourth quadrants, respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents to the parabola at P and Q intersect the x-axis at S.

[JEE 2007, 4 + 4 + 4]

(a) The ratio of the areas of the triangles PQS and PQR is

(A) 1 : $\sqrt{2}$ (B) 1 : 2 (C) 1 : 4 (D) 1 : 8

Sol.

(b) The radius of the circumcircle of the triangle PRS is

(A) 5 (B) $3\sqrt{3}$ (C) $3\sqrt{2}$ (D) $2\sqrt{3}$

Sol.

(c) The radius of the incircle of the triangle PQR is

(A) 4 (B) 3 (C) 8/3 (D) 2

Sol.

13. The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose

(A) vertex is (2a/3 ,0) (B) directrix is x = 0
(C) latusrectum is 2a/3 (D) focus is (a, 0)

[JEE 2009]

Sol.

14. Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be **[JEE 2010]**

(A) $-\frac{1}{r}$ (B) $\frac{1}{r}$ (C) $\frac{2}{r}$ (D) $-\frac{2}{r}$

Sol.

15. Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latus rectum and the point $P\left(\frac{1}{2}, 2\right)$ on the parabola, and Δ_2 be the area of the triangle formed by drawing tangents at P and at the end points of the latus rectum.

Then $\frac{\Delta_1}{\Delta_2}$ is **[JEE 2011]**

Sol.

16. Let (x, y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from (0, 0) to (x, y) in the ratio 1 : 3. Then the locus of P is

(A) $x^2 = y$ (B) $y^2 = 2x$ (C) $y^2 = x$ (D) $x^2 = 2y$

Sol. **[JEE 2011]**

17. Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point (9, 6), then L is given by

(A) $y - x + 3 = 0$ (B) $y + 3x - 33 = 0$

(C) $y + x - 15 = 0$ (D) $y - 2x + 12 = 0$

Sol. **[JEE 2012]**